WEAKLY NONLINEAR DISTURBANCES IN CONCENTRATED GAS SUSPENSIONS

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The propagation of linear acoustical disturbances in gas suspensions containing solid particles has been investigated repeatedly (for example, see [1], as well as [2-4]), with the speed of sound being determined in the majority of cases using the equilibrium equation of state for the mixture as a whole. Attempts to allow for nonequilibrium processes of dynamic and thermal relaxation, usually made at the phenomenological level through the introduction of the appropriate relaxation times, are contained in [2-6], for example. Recently the problem of obtaining a single evolutionary equation for waves of small but finite amplitude has also been considered; different versions of the Burgers equation have been obtained as a result [5-7].

A significant defect of these reports is that the exchange of heat and momentum between phases is usually described within the framework of the simplest relaxation formalism, when one uses some a priori relations, the insufficient (and sometimes simply the unknown) accuracy of which restricts the actual limits of applicability of the final results obtained [5]. In addition, either dynamic or thermal relaxation is often neglected in order to make it possible to use either a one-velocity or a one-temperature model of a two-phase mixture [6, 7], and one also assumes that the volumetric concentration of particles in the mixture is low, and thereby neglects all constraint effects in describing processes of exchange between phases, the role of which is very large in concentrated systems.

In the theory proposed below these limitations are removed. Specifically, the propagation of waves of not too high frequency in moderately concentrated gas suspensions (with a volumetric concentration of suspended particles of up to 20-25%) is investigated.

1. Equations of Conservation and Exchange between Phases

Let us consider one-dimensional disturbances in a quiescent, two-phase mixture consisting of a compressible ideal gas and inert, incompressible, solid spheres of the same small radius suspended in it. For the continuous and disperse phases in the continuous approximation we write the equations of conservation of mass

$$\frac{\partial(\varepsilon d_0)}{\partial t}+\frac{\partial(\varepsilon d_0 u)}{\partial x}=0,\ \frac{\partial\rho}{\partial t}+\frac{\partial(\rho v)}{\partial x}=0,\ \varepsilon=1-\rho; \tag{1.1}$$

conservation of momentum

$$\varepsilon d_0\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}\right)u=-\frac{\partial p}{\partial x}-f,\ \rho d_1\left(\frac{\partial}{\partial t}+v\frac{\partial}{\partial x}\right)v=f; \tag{1.2}$$

and heat flow

$$\begin{gathered}\varepsilon d_0 C_V\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}\right)T_0+Rd_0T_0\frac{\partial}{\partial x}(\varepsilon u+\rho v)=-Q+(u-v)f,\\ \rho d_1 C_1\left(\frac{\partial}{\partial t}+v\frac{\partial}{\partial x}\right)T_1=Q,\end{gathered} \tag{1.3}$$

where d_0, d_1, C_V, and C_1 are the densities and specific heats of the materials of the phases; u, v, T_0, and T_1, average velocities and temperatures; f and Q, fluxes of momentum and heat between phases calculated per unit volume of the mixture; ρ, volumetric concentration of particles; R, universal gas constant, normalized to the molecular weight of the gas. The equations for the continuous phase in (1.1) and (1.2), the equation of state of an ideal

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gas, and the well-known thermodynamic relations were used in the derivation of the first equation in (1.3). The molecular viscosity and heat conduction are taken into account in (1.2) and (1.3) only in determining quantities characterizing the exchange between phases.

To close the system (1.1)-(1.3) we need concrete expressions for the force f and the heat flux Q, which can be obtained in principle from an analysis of nonsteady dynamic and thermal problems for an individual particle with allowance for the influence of all other particles. Such problems were solved in [8, 9] for low values of the Reynolds and Peclet numbers and for moderately concentrated disperse systems (in which the nonoverlapping of particles can be neglected). In general, the expressions for f and Q are very cumbersome; in the concrete calculations below we use their low-frequency limits, assuming in addition that the ratio of thermal conductivities of the particle material and the gas is far greater than unity. In this case from [8] we have

$$f \approx \frac{9}{2}\frac{\rho}{1-5\rho/2}\frac{\mu_0}{a^2}\left[1+O\left(\sqrt{\frac{\omega a^2}{\nu_0}}\right)\right](u-v),$$

where a is the particle radius; μ_0 and ν_0, dynamic and kinematic viscosities; ω, characteristic frequency of the process. From this and from the second equation in (1.2) we get

$$u \approx \left(1+\tau_d\frac{\partial}{\partial t}\right)v, \quad \tau_d = \frac{2}{9}\left(1-\frac{5}{2}\rho\right)\frac{a^2}{\nu_0}\frac{d_1}{d_0}. \qquad (1.4)$$

From [9] we obtain

$$T_1 \approx \left(1-\tau_t\frac{\partial}{\partial t}\right)T_0, \quad \tau_t = \frac{1-3\rho}{3}\left[1-\frac{9}{5}\left(\rho+\varepsilon\frac{d_0C_p}{d_1C_1}\right)\right]\frac{a^2}{\varkappa_0}\frac{d_1C_1}{d_0C_p}, \qquad (1.5)$$

where $\varkappa_0$ is the thermal diffusivity of the gas. The specific heat of the gas at constant pressure is analyzed in (1.5) in accordance with [10]. It is seen that for real gas suspensions the times of dynamic and thermal relaxation, τ_d and τ_t, introduced above have the same order of magnitude. Equations (1.4) and (1.5) are valid when the inequalities

$$\omega \ll \nu_0/a^2, \quad \omega \ll \varkappa_0/a^2 \approx \varkappa_1/a^2$$

are satisfied; when they are violated one must allow for terms in the expression for the force f which correspond to the Basset and Faxen forces, as well as the buoyant force in the field of inertial forces [8], while in Eq. (1.5) one must allow for terms containing higher time derivatives. The above-indicated inequalities are assumed to be satisfied; since $d_1/d_0 \simeq 10^3$, however, even under this condition the quantities $\omega\tau_d$ and $\omega\tau_t$ can have the order of 10^2. We also note that the upper limit on the frequency ω follows from the condition of adequacy of the continuous approximation: The length of a sound wave must be far greater than the spatial microscale of the mixture.

2. Speed of Sound

Denoting the quantities in a quiescent gas suspension by the adopted symbols and their perturbations by the same symbols with a prime, from (1.1)-(1.5) we obtain the following system of linearized equations:

$$\begin{gathered}
\varepsilon\frac{\partial d_0'}{\partial t}+d_0\frac{\partial v'}{\partial x}+\varepsilon d_0\tau_d\frac{\partial^2 v'}{\partial t\partial x}=0,\\
d\frac{\partial v'}{\partial t}+\varepsilon d_0\tau_d\frac{\partial^2 v'}{\partial t^2}+R\left(d_0\frac{\partial T_0'}{\partial x}+T_0\frac{\partial d_0'}{\partial x}\right)=0,\\
dC\frac{\partial T_0'}{\partial t}-\rho d_1C_1\tau_t\frac{\partial^2 T_0'}{\partial t^2}+Rd_0T_0\left(\frac{\partial v'}{\partial x}+\varepsilon\tau_d\frac{\partial^2 v'}{\partial t\partial x}\right)=0.
\end{gathered} \qquad (2.1)$$

Here we introduce the average density and the average specific heat of the mixture,

$$d = \varepsilon d_0+\rho d_1, \quad dC = \varepsilon d_0C_V+\rho d_1C_1. \qquad (2.2)$$

Let us consider a plane monochromatic wave characterized by the variable multiplier

$$\exp[i(kx - \omega t)] = \exp\{(\omega/c)[i(x - ct) - mx]\}, \quad \operatorname{Im}\omega = 0, \quad \operatorname{Re}k = \omega/c, \quad \operatorname{Im}k = \omega m/c, \tag{2.3}$$

where c and m are the velocity of propagation of the wave and its effective attenuation coefficient.

Substituting (2.3) into (2.1), from the characteristic equation of the resulting system of linear equations for the amplitudes d_0', v' and T_0' we have the dispersion relation

$$\left(\frac{k}{\omega}\right)^2 = \frac{\varepsilon(d - i\varepsilon d_0\omega\tau_d)(dC + i\rho d_1C_1\omega\tau_t)}{Rd_0T_0(1 - i\varepsilon\omega\tau_d)(dC + \varepsilon d_0R + i\rho d_1C_1\omega\tau_t)}. \tag{2.4}$$

Separating the real and imaginary parts here and using the definitions of d and dC in (2.2) and of c and m in (2.3), we obtain

$$c = \frac{c_0}{F}\left(1 + \frac{1}{\sqrt{1+\alpha^2}}\right)^{-1/2}, \quad m = \frac{\alpha}{1 + \sqrt{1+\alpha^2}}, \tag{2.5}$$

where

$$F = \left[\frac{\varepsilon\gamma}{2}\frac{\sqrt{(AA' + BB')^2 + (AB' - A'B)^2}}{A'^2 + B'^2}\right]^{1/2};$$

$$\alpha = \frac{AB' - A'B}{AA' + BB'}; \quad \gamma = \frac{C_p}{C_V}; \quad c_0^2 = \gamma RT_0;$$

$$A = (\varepsilon + \rho\Lambda_d)(\varepsilon + \rho\Lambda_t) + \rho\varepsilon\Lambda_t\omega\tau_d\omega\tau_t;$$

$$A' = \varepsilon\gamma + \rho\Lambda_t + \rho\varepsilon\Lambda_t\omega\tau_d\omega\tau_t;$$

$$B = \varepsilon(\varepsilon + \rho\Lambda_t)\omega\tau_d - \rho\Lambda_t(\varepsilon + \rho\Lambda_d)\omega\tau_t;$$

$$B' = \varepsilon(\varepsilon\gamma + \rho\Lambda_t)\omega\tau_d - \rho\Lambda_t\omega\tau_t;$$

$$\Lambda_d = d_1/d_0; \quad \Lambda_t = d_1C_1/d_0C_V.$$

Equations (2.5) differ quite significantly from the known equations. If we even assume, as is done in [6, 7], that $\tau_d \approx 0$ and $\tau_t \neq 0$ (i.e., that dynamic relaxation takes place considerably faster than thermal relaxation) and we consider the low-frequency limit $\omega\tau_t \ll 1$, from (2.5) we obtain for the speed of sound of the wave

$$\frac{c}{c_0} \approx \left[\frac{\gamma\varepsilon + \rho\Lambda_t}{\varepsilon\gamma(\varepsilon + \rho\Lambda_t)(\varepsilon + \rho\Lambda_d)}\right]^{1/2}. \tag{2.6}$$

According to [6], where they used similar assumptions and started directly from a one-velocity model of the medium, we have

$$\frac{c}{c_0} = \left[\frac{\gamma + \rho\Lambda_t}{\gamma(1 + \rho\Lambda_t)}\right]^{1/2}.$$

It is seen that for $\rho\Lambda_d \gtrsim 1$ (i.e., $\rho \gtrsim 10^{-3}$) the latter result is greatly overstated.

The concentration of the mixture affects the propagation velocity and attenuation coefficient of sound waves, first, because the effective density and specific heat of the mixture depend on it, and second, because of the dependence on it of the characteristic times of dynamic and thermal relaxation, which reflects the influence of constraint on the processes of exchange between phases. The first dependence becomes important even for very dilute gas suspensions, while the second is important only for gas suspensions of sufficiently high concentration.

Specific calculations from Eqs. (2.5) were made for a suspension of quartz sand in air at atmospheric pressure and a temperature of about 20°C. In this case we have approximately $\Lambda_d \approx \Lambda_t \approx 1.77 \cdot 10^3$, $\gamma \approx 1.40$, and a ratio of relaxation times

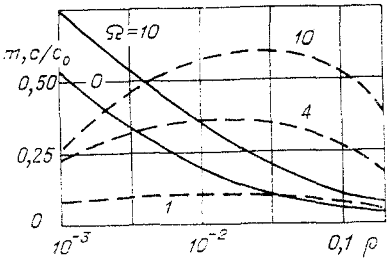

Fig. 1

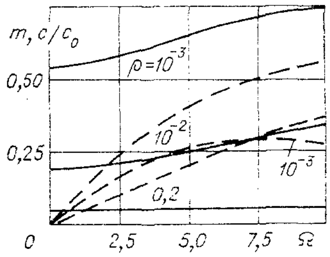

Fig. 2

$$\frac{\tau_t}{\tau_d} \approx 0.78\,\frac{1-3\rho}{1-5\rho/2}\left[1-1.8\left(\rho+\frac{\varepsilon}{1.26\cdot 10^3}\right)\right].$$

Then we have

$$\tau_d\omega = \frac{2}{9}\left(1-\frac{5}{2}\rho\right)\Omega, \quad \tau_t\omega = \frac{2}{9}\left(1-\frac{5}{2}\rho\right)\frac{\tau_t}{\tau_d}\Omega, \quad \Omega = \frac{d_1}{d_0}\frac{a^2}{\nu_0}\omega.$$

In Fig. 1 we present c/c_0 and m (solid and dashed curves, respectively) as functions of ρ for different Ω (numbers on curves), and in Fig. 2 as functions of Ω for different ρ. It is seen that the attenuation of sound waves is small only for very low mixture concentrations or frequencies. For $\rho \gtrsim 0.01$ and $\Omega \gtrsim 1$ significant attenuation occurs even over distances on the order of a wavelength.

3. Evolutionary Equation

Now let us consider the propagation of disturbances of finite amplitude, slightly nonlinear in the sense that distortions of the profile of the disturbance caused both by the nonlinearity and by dissipation are relatively small over distances on the order of a wavelength. As follows from the foregoing, such an assumption can be proper only for sufficiently small ρ and ω; therefore, here we are confined to the analysis of the situation when $\omega\tau_d \approx \omega\tau_t \ll 1$. A wave with little distortion of the profile can be analyzed using the well-known method of many scales. In the present case this comes down to the fact that its amplitude, multiplied by the variable quantity in (2.3), is assumed to depend on the product $z = \delta x$, where δ is a quantity of first-order smallness with respect to the amplitude (see [11, 12], for example).

We introduce the new variables

$$y = t - x/c, \quad z = \delta x, \tag{3.1}$$

where the speed c is defined in (2.6), corresponding to a comoving coordinate system for low-frequency waves of small amplitude.

To within terms of second order with respect to amplitude, inclusively, from (1.1)-(1.3) we obtain a system of equations replacing the linear equations in (2.1), which in the comoving coordinate system (3.1) is written in the form

$$\begin{gathered} d_0\frac{\partial v'}{\partial z} + \varepsilon\frac{\partial d_0}{\partial y} - \frac{d_0}{c}\frac{\partial v'}{\partial y} - \frac{1}{c}\frac{\partial\left(v'd_0'\right)}{\partial y} - \varepsilon d_0\frac{\tau_d}{c}\frac{\partial^2 v'}{\partial y^2} = 0, \\ \left(\varepsilon + \frac{d_1}{d_0}\rho + \varepsilon\frac{d_0'}{d_0} - \frac{v'}{c}\right)\frac{1}{c}\frac{\partial v'}{\partial y} + \varepsilon\frac{\tau_d}{c}\frac{\partial^2 v'}{\partial y^2} + RT_0\left[\frac{1}{cd_0}\frac{\partial d_0'}{\partial z} + \frac{1}{cT_0}\frac{\partial T_0'}{\partial z}\right. \\ \left. - \frac{1}{c^2 d_0}\frac{\partial d_0}{\partial y} - \frac{1}{c^2 T_0}\frac{\partial T_0'}{\partial y} - \frac{1}{c^2 d_0 T_0}\frac{\partial\left(d_0'T_0'\right)}{\partial y}\right] = 0, \\ \left(\varepsilon + \rho\frac{d_1 C_1}{d_0 C_V} + \varepsilon\frac{d_0'}{d_0} - \frac{v'}{c}\right)\frac{1}{T_0}\frac{\partial T_0'}{\partial y} - \frac{d_1 C_1}{d_0 C_V}\rho\frac{\tau_t}{T_0}\frac{\partial^2 T_0'}{\partial y^2} + (\gamma-1)\left(\frac{\partial v'}{\partial z} - \frac{1}{c}\frac{\partial v'}{\partial y} - \varepsilon\frac{\tau_d}{c}\frac{\partial^2 v'}{\partial y^2} - \frac{1}{c}\frac{d_0'}{d_0}\frac{\partial v'}{\partial y} - \frac{1}{c}\frac{T_0'}{T_0}\frac{\partial v'}{\partial y}\right) = 0. \end{gathered} \tag{3.2}$$

From the linear problem discussed in Sec. 2 we can obtain, in the low-frequency limit $\omega\tau_d \approx \omega\tau_t \ll 1$, the following relations, valid to within quantities of first order with respect to amplitude:

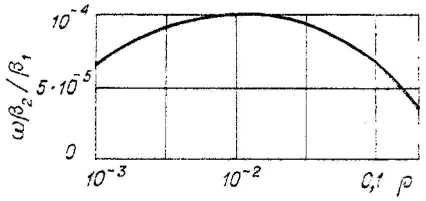

Fig. 3

$$\frac{v'}{c}=\varepsilon\frac{d_0'}{d_0}=\frac{\varepsilon+\rho\Lambda_t}{\gamma-1}\frac{T_0'}{T_0}=\frac{\rho'}{\rho}. \tag{3.3}$$

Transforming terms of second-order smallness in (3.2) using (3.3), which does not impair the accuracy adopted here, multiplying the first equation in (3.2) by $RT_0/\varepsilon c^2 d_0$ and the third by $(RT_0/c^2)(\varepsilon + \rho\Lambda_t)^{-1}$, and combining the results with the second equation, after calculations we obtain a single evolutionary equation for slightly nonlinear waves in the form of the Burgers equation

$$\frac{\partial v'}{\partial z}=\frac{\beta_1}{c^2}v'\frac{\partial v'}{\partial y}+\frac{\beta_2}{c}\frac{\partial^2 v'}{\partial y^2}. \tag{3.4}$$

Here we introduce the coefficients

$$\beta_1=\frac{1}{2}\left[\frac{2}{\varepsilon}+\frac{\gamma-1}{\varepsilon+\rho\Lambda_t}\left(3+\frac{\varepsilon}{\varepsilon+\rho\Lambda_t}\right)\right]\left(1+\frac{\varepsilon(\gamma-1)}{\varepsilon+\rho\Lambda_t}\right)^{-1},$$
$$\beta_2=\frac{1}{2}\left\{\varepsilon\left[1-\varepsilon\gamma\left(\frac{c}{c_0}\right)^2+\frac{\varepsilon(\gamma-1)}{\varepsilon+\rho\Lambda_t}\right]\tau_d+\frac{\varepsilon(\gamma-1)\rho\Lambda_t}{(\varepsilon+\rho\Lambda_t)^2}\tau_t\right\}\left(1+\frac{\varepsilon(\gamma-1)}{\varepsilon+\rho\Lambda_t}\right)^{-1}. \tag{3.5}$$

A similar equation can also be obtained [using (3.2] for any other quantity that varies in the wave. The Burgers equation has been investigated in detail earlier (see [11, 12], for example).

If the concentration ρ is not too low, from (3.5) we get

$$\beta_1\approx\frac{1}{\varepsilon},\quad \beta_2\approx\frac{1}{2}\varepsilon\left[1-\varepsilon\gamma\left(\frac{c}{c}\right)^2\right]\tau_d\approx\frac{1}{2}\varepsilon\tau_d,$$

from which it is seen that whereas the characteristics of linear acoustical disturbances are affected equally strongly by processes of dynamic and thermal relaxation, the "relaxation viscosity," counteracting the nonlinear "steepening" of slightly nonlinear waves, is determined mainly by dynamic relaxation. Therefore, in particular, one can approximately analyze the propagation of such waves using a two-velocity, one-temperature model of the two-phase mixture, but a one-velocity, two-temperature model proves inadequate.

The relation between the orders of magnitude of the two terms on the right side of Eq. (3.4) is determined by the value of the parameter $\omega\beta_2/\beta_1$, where ω is the characteristic wave frequency, as before. The dependence of this parameter on ρ for $\Omega = 10^{-3}$ is presented in Fig. 3; such dependences corresponding to different Ω, like the analogous dependences of the attenuation coefficient m, have maxima at $p \leqslant 0.01$. At the corresponding concentrations the relative effects of dissipation are particularly strong.

LITERATURE CITED

1. A. N. Kraiko, R. I. Nigmatulin, V. K. Starkov, and L. E. Sternin, "The mechanics of multiphase media," Gidromekhanika, 6 (1972).
2. P. P. Zolotarev, "Propagation of weak disturbances in mixtures," Izv. Akad. Nauk SSSR, Mekh. Mashinostr., No. 4 (1964).
3. V. S. Popov, "Propagation of small disturbances in gases containing solid particles in a suspended state," Inzh.-Fiz. Zh., 14, No. 4 (1968).
4. F. E. Marble, "Dynamics of dusty gases,"Ann. Rev. Fluid Mech., 2 (1970).
5. G. A. Davidson, "A Burgers' equation approach to finite amplitude acoustics in aerosol media," J. Sound Vib., 38, No. 4 (1975).
6. A. A. Borisov, A. F. Vakhgel't, and V. E. Nakoryakov, "Propagation of long-wavelength disturbances of finite amplitude in gas suspensions," Zh. Prikl. Mekh. Fiz., No. 5 (1980).

7. A. A. Borisov and A. F. Vakhgel't, "Wave processes in two-phase media containing solid particles," in: Wave Processes in Two-Phase Media [in Russian], Inst. Teplofiz. Sib. Otd. Akad. Nauk SSSR, Novosibirsk (1980).
8. Yu. A. Buevich and V. G. Markov, "Rheological properties of finely dispersed suspensions. Nonsteady flows," Inzh.-Fiz. Zh., 34, No. 6 (1978).
9. Yu. A. Buevich and Yu. A. Korneev, "The dispersion of thermal waves in granular material," Inzh.-Fiz. Zh., 31, No. 1 (1976).
10. L. D. Landau and E. M. Lifshits, The Mechanics of Continuous Media [in Russian], Gostekhteorizdat, Moscow (1954).
11. O. V. Rudenko and S. I. Soluyan, Theoretical Principles of Nonlinear Acoustics [in Russian], Nauka, Moscow (1975).
12. G. B. Whitham, Linear and Nonlinear Waves, Wiley, New York (1974).

STABILITY OF COLLIDING DROPS OF IDEAL LIQUID

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The determination of the stability conditions of a system of colliding drops is of interest not only as one of the classical problems of fluid mechanics, but is urgent for the solution of a number of applied problems. A rather detailed analysis of the work on this problem performed up to 1970 is given in [1]. In the last decade interest in the physics of the interaction of drops has been stimulated by the development of the gasdynamics of two-phase flows in jets. So far, however, the laws of collision, deformation, coalescence, and disruption of drops have not been conclusively determined.

In the present article we present the results of an experimental and analytical study of the stability of a system of two colliding drops of an ideal liquid. To describe the interaction of drops quantitatively we use the following dimensionless numbers obtained by similitude theory and dimensional analysis [2, 3]:

$$\gamma = D_2/D_1 \tag{1}$$

is the ratio of the diameters of the drops, $We = \rho u^2 D_1/\sigma$ is the Weber number, $\Omega = |M|/[(D_0/2)^{7/2}\sqrt{\rho\sigma}]$ is the normalized angular momentum. Here

$$\mathbf{M} = m_1 m_2 u\delta/(m_1 + m_2), \quad D_0 = (D_1^3 + D_2^3)^{1/3}, \tag{2}$$

where m_1 and m_2 are the masses of the colliding drops, δ is the impact parameter, and M is the angular momentum of the system of drops.

Since for water drops viscous forces are negligibly small in comparison with surface tension and inertial forces, the effect of dimensionless numbers involving the viscosity (e.g., $Lp = \rho\sigma D_2/\eta^2 \sim 10^5$, $Re = \rho u D_1/\eta \sim 10^3$) is unimportant.

An experimental study of the types of interaction of water drops for We = 0.1-120 (γ = 1.9) showed that for We = 15-50 the interaction is characterized by coalescence of the drops with a subsequent possible disruption under the action of centrifugal forces [2]. Consequently, it is expedient to seek the limit of stability of a system of drops in this range of Weber numbers. We have investigated stability conditions of a system of two colliding drops for γ = 1.15-2.6 and We = 10-50. The apparatus (Fig. 1) consisted of two generators 1 producing counterstreams of water drops whose diameters could be varied from 0.3×10^{-3} to 1.2×10^{-3} m. The density, dynamic viscosity, and surface tension of the drops of distilled water were $\rho = 10^3$ kg/m³, $\eta = 10^{-3}$ kg/m · sec, and $\sigma = 73 \times 10^{-3}$ kg/sec² at 20°C. The relative velocities u of the colliding drops varied from 1 to 5 m/sec. Three-dimensional photographs were taken two SKS-1 m motion-picture cameras 2 located at right angles to one another and perpendicular to the streams of drops. Illumination was provided by photoflood lamps 3

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